



$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\psi_A(A) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\psi_D(D) = \psi_B(B) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\psi_C(C) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Whenever

$$\psi_{ij}(x_i, x_j) = \left\{ \begin{array}{c|cc} & x_i & x_j \\ \hline 1 & 0 & 0 \\ 2 & 0 & 1 \\ 3 & 1 & 0 \\ 4 & 1 & 1 \end{array} \right\}$$

$$\psi_{ij}(0, 0) = 1$$

$$\psi_{ij}(1, 1) = 4 \dots$$

Loopy BP:

$$1. M_{i \rightarrow j}(x_j) = \frac{1}{2} \quad \forall i$$

Example loop 1 message:

$$2. M_{D \rightarrow A}(X_A) = \sum_{X_D} \psi_D(X_D) \psi_{DA}(X_D, X_A) \cdot M_{C \rightarrow D}(X_D)$$

$$\begin{array}{l} X_D = 0 \\ X_D = 1 \end{array}$$

$$= 1 \cdot \psi_{DA}(0, X_A) \cdot M_{C \rightarrow D}(0) +$$

$$+ 1 \cdot \psi_{DA}(1, X_A) \cdot M_{C \rightarrow D}(1) =$$

$$= \begin{pmatrix} 1 \cdot \psi_{DA}(0,0) \cdot \frac{1}{2} + 1 \cdot \psi_{DA}(1,0) \cdot \frac{1}{2} \\ 1 \cdot \psi_{DA}(0,1) \cdot \frac{1}{2} + 1 \cdot \psi_{DA}(1,1) \cdot \frac{1}{2} \end{pmatrix} =$$

$$= \begin{pmatrix} 1 \cdot 1 \cdot \frac{1}{2} + 1 \cdot 3 \cdot \frac{1}{2} \\ 1 \cdot 2 \cdot \frac{1}{2} + 1 \cdot 4 \cdot \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\propto \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$$

Example loop 2 message: From Loop 1



$$M_{A \rightarrow B}(X_B) = \sum_{X_A} \psi_A(X_A) \psi_{AB}(X_A, X_B) M_{D \rightarrow A}(X_A)$$

$$= \psi_A(0) \psi_{AB}(0, X_B) M_{D \rightarrow A}(0) +$$

$$+ \psi_A(1) \psi_{AB}(1, X_B) M_{D \rightarrow A}(1)$$

$$= 1 \psi_{AB}(0, X_B) \cdot 0.4 +$$

$$+ 2 \psi_{AB}(1, X_B) \cdot 0.6 =$$

$$X_B=0 \left(0.4 \cdot \psi_{AB}(0,0) \cdot 1 + 2 \cdot 0.6 \cdot \psi_{AB}(1,0) \right)$$

$$X_B=1 \left(0.4 \cdot \psi_{AB}(0,1) \cdot 1 + 2 \cdot 0.6 \cdot \psi_{AB}(1,1) \right)$$

$$= \begin{pmatrix} 0.4 \cdot 1 \cdot 1 + 2 \cdot 0.6 \cdot 3 \\ 0.4 \cdot 2 \cdot 1 + 2 \cdot 0.6 \cdot 4 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 5.6 \end{pmatrix} \propto \begin{pmatrix} 0.42 \\ 0.58 \end{pmatrix}$$

$$\psi_A(X_A) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$