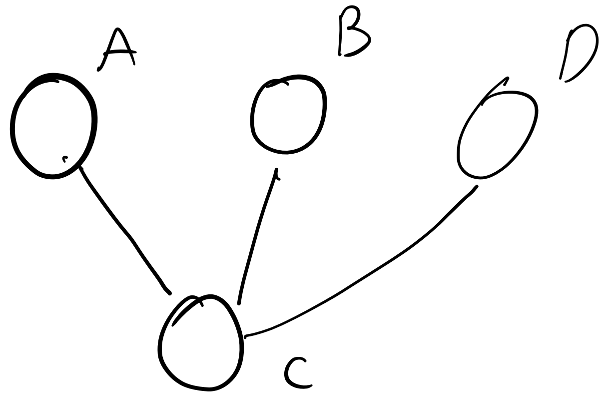


A, B, C, D binary

$\{0, 1\}$

$$\psi_{ij}\left(x_i, x_i\right) = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix} \forall i, j$$

X_i, X_j the same

X_i, X_j different

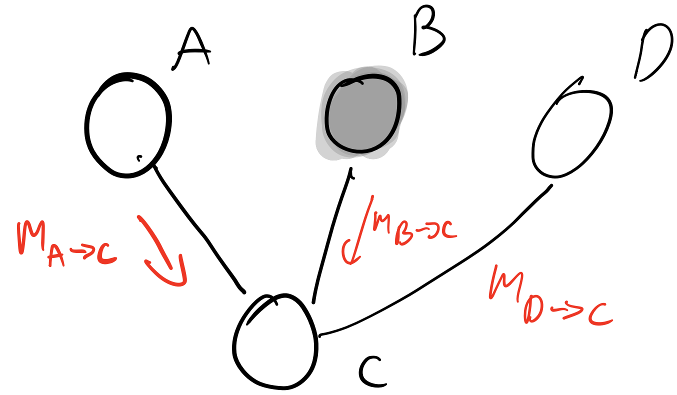
"more 0 than 1"

$$\psi_A = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \psi_B = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \psi_C = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$\psi_D = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$ "more 1 than 0"

Want Marginals for A, D.
B was observed as 1

1. Pick root → C
2. Pass from leaves to root



$$M_{A\to c}(x_c) = \sum_{A} \psi_A(x_A)\,\psi_{Ac}(x_A)$$

$$= 1 \cdot \begin{pmatrix} 5 \\ 1 \end{pmatrix} + 1 \cdot \begin{pmatrix} 1 \\ 5 \end{pmatrix} =$$

$$= \begin{pmatrix} 6 \\ 6 \end{pmatrix} = \begin{pmatrix} M_{A\to c}(0) \\ M_{A\to c}(1) \end{pmatrix}$$

$$M_{B\to c}(c) = \psi_B(\overline{x}_B)\,\psi_{BC}(\overline{x}_B, x_c)$$

$$= 1 \cdot \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$\psi_{BC}(1,0) = 1$

$\psi_{BC}(1,1) = 5$

$\psi_B(1) = 1$

$M_{B\to c}(0) = 1$

$M_B \to c(1) = 5$

"I'm a 1 and we should be like eachother so you shold also be more like a 1"

$$M_{D\to c}(C) = \sum_D \psi_D(x_D)\,\psi_{DC}(X_D, x_c)$$

$$= 2 \cdot \begin{pmatrix} 1 \\ 5 \end{pmatrix} + 1 \cdot \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 7 \\ 11 \end{pmatrix}$$

$$\psi_D(1) = 2 > 1 = \psi_D(0) \qquad \psi_{DC}(x_D, x_C)$$

"I'm more 1 than 0 and we should be more like each other so you should be more 1 than 0

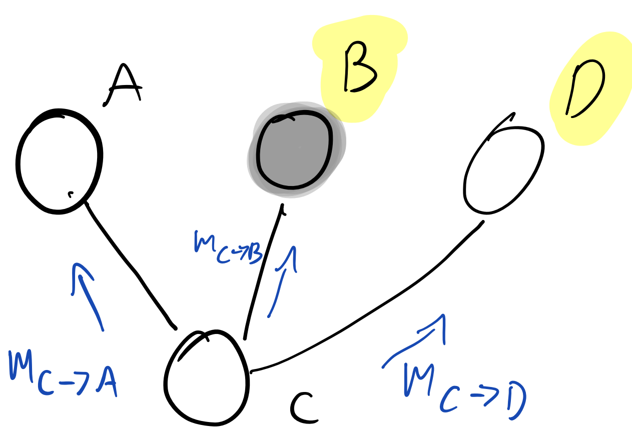
$$M_{D\to C}(1) = 11 > 7 = M_{D\to C}(0)$$

Send from root to leaves

$$m_{A\to C} = \begin{pmatrix}6\\6\end{pmatrix}$$

$$m_{B\to C} = \begin{pmatrix}1\\5\end{pmatrix}$$

$$m_{D\to C} = \begin{pmatrix}7\\11\end{pmatrix}$$



$$M_{C \to A}(X_A) = \sum_{C} \psi_C(X_C)\, \psi_{CA}(X_C, X_A) \prod_{k \in N(C)/A} m_{k \to C}(X_C)$$

$N(C)/A = \{B, D\}$

$$= \sum_{C} \psi_C(X_C)\, \psi_{CA}(X_C, X_A)\, m_{B \to C}(X_C)\, m_{D \to C}(X_C)$$

$$= \underbrace{5 \cdot \begin{pmatrix} 5 \\ 1 \end{pmatrix} \cdot 1 \cdot 7}_{C=0} + \underbrace{1 \cdot \begin{pmatrix} 1 \\ 5 \end{pmatrix} \cdot 5 \cdot 11}_{C=1}$$

$$\begin{array}{c} A=0 \\ = \\ A=1 \end{array} \begin{pmatrix} 175 \\ 35 \end{pmatrix} + \begin{pmatrix} 55 \\ 275 \end{pmatrix} = \begin{pmatrix} 230 \\ 310 \end{pmatrix}$$

$$M_{C\to D}(X_D) = \sum_{C} \psi_C(X_C)\, \psi_{CD}(X_C, X_D)\, M_{A\to C}(X_C)\, M_{B\to C}(X_C)$$

$$= 5 \cdot \begin{pmatrix} 5 \\ 1 \end{pmatrix} \cdot 6 \cdot 1 + 1 \cdot \begin{pmatrix} 1 \\ 5 \end{pmatrix} \cdot 6 \cdot 5$$

$$= \begin{pmatrix} 150 \\ 30 \end{pmatrix} + \begin{pmatrix} 30 \\ 150 \end{pmatrix} = \begin{pmatrix} 180 \\ 180 \end{pmatrix}$$

we don't need $M_{C\to B}$ since we know B!

$$\tilde{b}(A) = \psi_A(X_A) \cdot M_{C\to A}(X_A) = \begin{pmatrix} 1 \cdot 230 \\ 1 \cdot 310 \end{pmatrix}$$

$$\tilde{b}(C) = \psi_C(X_C) \cdot M_{A\to C}(X_C)\, M_{B\to C}(X_C)\, M_{D\to C}(X_C)$$

$$= \begin{pmatrix} 5 \cdot 6 \cdot 1 \cdot 7 \\ 1 \cdot 6 \cdot 5 \cdot 11 \end{pmatrix} = \begin{pmatrix} 210 \\ 330 \end{pmatrix}$$

$$\tilde{b}(D) = \psi_D(X_D) \cdot \mu_{C \to D}(X_D)$$

$$= \begin{pmatrix} 1 \cdot 180 \\ 2 \cdot 180 \end{pmatrix} = \begin{pmatrix} 180 \\ 360 \end{pmatrix}$$

$$b(A) = \begin{pmatrix} 230 \\ 310 \end{pmatrix} \cdot \frac{1}{230+310} \approx \begin{pmatrix} 43\% \\ 57\% \end{pmatrix}$$

$$b(C) = \begin{pmatrix} 210 \\ 330 \end{pmatrix} \cdot \frac{1}{210+330} \approx \begin{pmatrix} 38\% \\ 62\% \end{pmatrix}$$

$$b(D) = \begin{pmatrix} 180 \\ 360 \end{pmatrix} \cdot \frac{1}{180+360} \approx \begin{pmatrix} 33\% \\ 67\% \end{pmatrix}$$